

Apple iPod

Creating Mathematical Models

“When am I ever going to use this?”
Using the concepts in this worksheet, you will be able to create mathematical models to represent real world contexts.

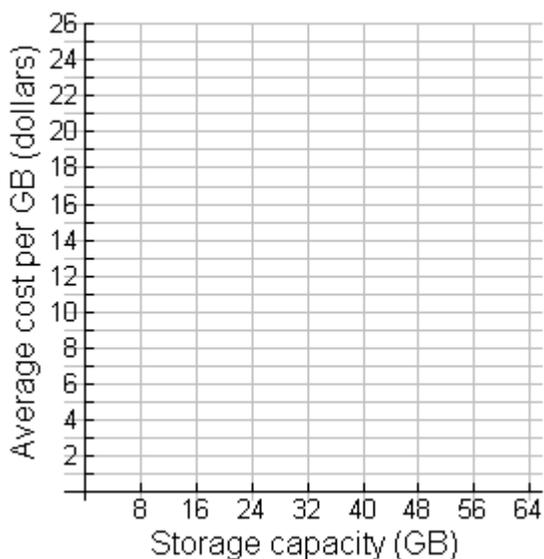
The Apple iPod, a portable music player, is available with a variety of storage capacities at different prices. The table below shows the price and storage capacity as of 2010 (Source: www.apple.com).

Price	Storage Capacity	Song Capacity	Video Capacity
\$199	8 GB	1750 songs	10 hours
\$299	32 GB	7000 songs	40 hours
\$399	64 GB	14,000 songs	80 hours

1. Create a mathematical model for song capacity as a function of video capacity. Then predict the number of songs that is equivalent to 25 hours of video.

2. Use a system of linear equations to create a quadratic function model for price as a function of storage capacity. Then forecast the price for an iPod with 16 GB of storage capacity. Consistent with the Apple marketing strategy, round the price to the nearest whole number ending with a 9.

3. Graph the average cost per GB of storage as a function of the storage capacity (in GB) on the axes below. Connect the plotted points with a smooth curve. Then use the curve to predict the average cost per GB of storage for an iPod with 16 GB of storage space.



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1. Create a mathematical model for song capacity as a function of video capacity. Then predict the number of songs that is equivalent to 25 hours of video.

To determine what type of function may best fit the data, we calculate the average rate of change for consecutive data points. We use the following data points for the analysis: (10,1750), (40,7000) and (80,14000).

$$\frac{14,000 - 7000}{80 - 40} \frac{\text{songs}}{\text{hours of video}}$$
$$= \frac{7000}{40}$$

$$= 175 \text{ songs per hour of video}$$

$$\frac{7000 - 1750}{40 - 10} \frac{\text{songs}}{\text{hours of video}}$$
$$= \frac{5250}{30}$$

$$= 175 \text{ songs per hour of video}$$

Since the average rate of change is constant, a linear function with slope 175 should be used to model the data.

$$y = mx + b$$

$$y = 175x + b$$

$$7000 = 175(40) + b$$

$$7000 = 7000 + b$$

$$b = 0$$

The linear model is $y = 175x$ where y is the number of songs and x is the number of hours of video. To predict the number of songs that is equivalent to 25 hours of video, we substitute $x = 25$ into the equation.

$$y = 175(25)$$

$$= 4375$$

Twenty-five hours of video is equivalent to 4375 songs on the iPod.

2. Use a system of linear equations to create a quadratic function model for price as a function of storage capacity. Then forecast the price for an iPod with 16 GB of storage capacity. Consistent with the Apple marketing strategy, round the price to the nearest whole number ending with a 9.

We use the following data points to create the model: $(8, 199)$, $(32, 299)$, and $(64, 399)$. A quadratic function model is of the form $y = ax^2 + bx + c$. Let p represent the price (in dollars) and s represent the storage capacity (in GB). Then the system of linear equations is

$$\begin{aligned} 199 &= a(8^2) + b(8) + c & 199 &= 64a + 8b + c \\ 299 &= a(32^2) + b(32) + c \text{ which simplifies to } & 299 &= 1024a + 32b + c \\ 399 &= a(64^2) + b(64) + c & 399 &= 4096a + 64b + c \end{aligned}$$

The system may be represented as an augmented matrix that may be written in reduced row echelon form using technology.

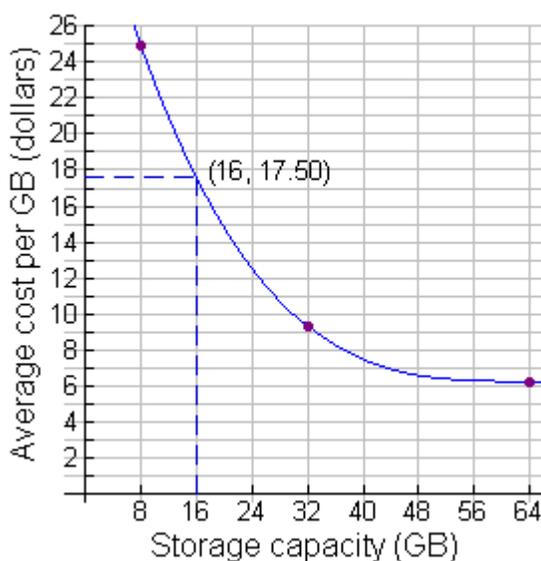
$$\left[\begin{array}{ccc|c} 64 & 8 & 1 & 199 \\ 1024 & 32 & 1 & 299 \\ 4096 & 64 & 1 & 399 \end{array} \right] \text{ reduces to } \left[\begin{array}{ccc|c} 1 & 0 & 0 & -0.01860 \\ 0 & 1 & 0 & 4.911 \\ 0 & 0 & 1 & 160.9 \end{array} \right]$$

The quadratic function model is $p = -0.0186s^2 + 4.911s + 160.9$. We evaluate this function at $s = 16$.

$$\begin{aligned} p &= -0.0186(16)^2 + 4.911(16) + 160.9 \\ &= 244.24 \end{aligned}$$

Rounding the price to the nearest number ending with a 9, we predict the price will be \$249.

3. Graph the average cost per GB of storage as a function of the storage capacity (in GB) on the axes below. Connect the plotted points with a smooth curve. Then use the curve to predict the average cost per GB of storage for an iPod with 16 GB of storage space.



The average cost per GB of storage for an iPod with 16 GB of storage is predicted to be approximately \$17.50.